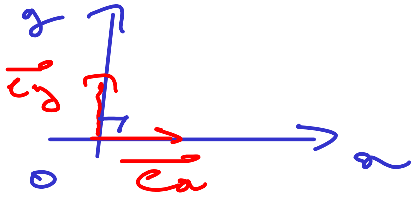
Vecteurs, base, projection ....

① Vecteurs de base ou base vectorielle orthonormée.

2-D.

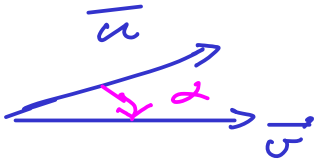


— normée : $\|\vec{e_x}\| = 1$; $\|\vec{e_y}\| = 1$ + φ : dim = ∅

— orthogonale : $\vec{e_x} \cdot \vec{e_y} = 0$ ou $\vec{e_x} \perp \vec{e_y}$

② Produit scalaire.

$\vec{u} \cdot \vec{v} = \underbrace{a}_{\in \mathbb{R}}$



$\alpha = (\vec{u}, \vec{v})$

⚠ angle orienté !

Calcul de a.

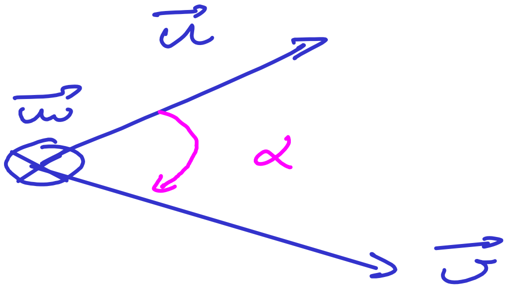
ⓐ $a = \|\vec{u}\| \|\vec{v}\| \cos\alpha$

ⓑ $\vec{u}\begin{pmatrix}u_1\\u_2\end{pmatrix}_{\mathcal{B}}$, $\vec{v} = \begin{pmatrix}v_1\\v_2\end{pmatrix}$

$a = u_1 v_1 + u_2 v_2$

③ Produit vectoriel.

$$\underbrace{\vec{u} \wedge \vec{v}}_{\text{« } \vec{u} \text{ vectoriel } \vec{v} \text{ »}} = \vec{w}$$



Calcul $\vec{w}$ ← norme, direction, sens

ⓐ
- norme : $\|\vec{w}\| = \|\vec{u}\|\,\|\vec{v}\|\,|\sin\alpha|$
- direction : $\vec{w} \perp \vec{u}$ et $\vec{w} \perp \vec{v}$
- sens : $\underset{p}{\vec{u}} \wedge \underset{i}{\vec{v}} = \underset{m}{\vec{w}}$) Règle des 3 doigts de la main droite.

ⓑ $\vec{u}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$ $\vec{v}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$

$$\vec{w} = \begin{pmatrix}u_1 & v_1\\ u_2 & v_2\\ u_3 & v_3\end{pmatrix} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ -(u_1v_3 - u_3v_1) \\ u_1v_2 - u_2v_1\end{pmatrix}$$

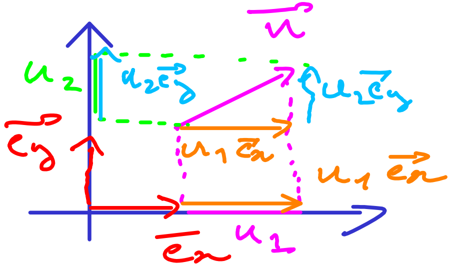
Rem :
- ⊙ vecteur ⊥ plan de la figure $\underbrace{\text{pointant vers nous}}_{\text{sortant}}$
- ⊗ // // // $\underbrace{\text{// vers la figure.}}_{\text{rentrant}}$

Application :

$\vec{e_x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\vec{e_y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\vec{e_z} = \vec{e_x} \wedge \vec{e_y} = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right)$

$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

1) Coordonnées d'un vecteur.



$\vec{u} \cdot \vec{e_x}$ $\vec{u} \cdot \vec{e_y}$

$\vec{u} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(\vec{e_x}, \vec{e_y})} \Longleftrightarrow \vec{u} = u_1\vec{e_x} + u_2\vec{e_y}$

notation condensée

on

$u_1 = \vec{u} \cdot \vec{e_x}$

$u_2 = \vec{u} \cdot \vec{e_y}$

Vérifions.

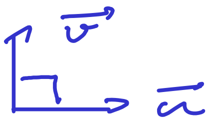
$\vec{u} \cdot \vec{e_x} = (u_1\vec{e_x} + u_2\vec{e_y}) \cdot \vec{e_x}$

$= u_1 \underbrace{\vec{e_x} \cdot \vec{e_x}}_{1} + u_2 \underbrace{\vec{e_y} \cdot \vec{e_x}}_{0}$

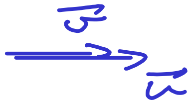
$= u_1$

5) Remarque

$\vec{u} \cdot \vec{v} = 0$? $\iff$ $\vec{u} \perp \vec{v}$

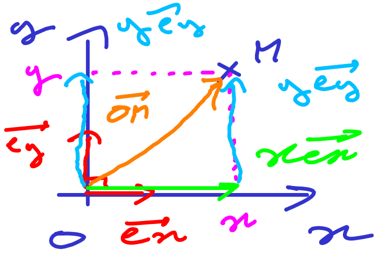


$\vec{u} \wedge \vec{v} = \vec{0}$? $\iff$ $\vec{u} \,/\!/\, \vec{v}$



ou

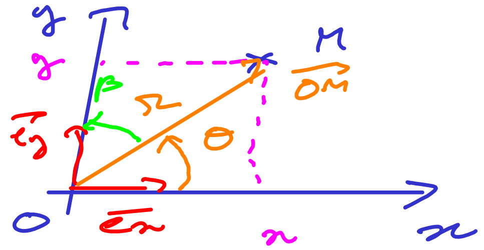
6) Vecteur position $\vec{OM}$



$$M\begin{pmatrix} x \\ y \end{pmatrix}_{(\vec{e_x},\,\vec{e_y})} \iff \vec{OM} = x\vec{e_x} + y\vec{e_y}$$

Par déf :
$$x = \vec{OM} \cdot \vec{e_x}$$
$$y = \vec{OM} \cdot \vec{e_y}$$

Application 1 :



x et y en fonction de r et θ ?

$$x = \overrightarrow{OM} \cdot \vec{e_x} = \underbrace{\|\overrightarrow{OM}\|}_{r} \underbrace{\|\vec{e_x}\|}_{1} \cos\theta$$

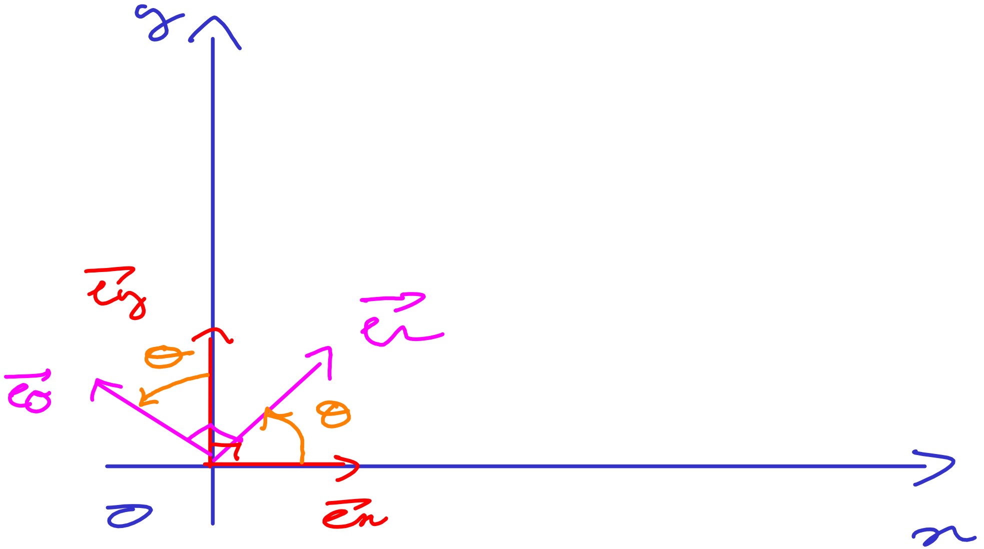
$$x = r\cos\theta$$

$$y = \overrightarrow{OM} \cdot \vec{e_y} = \|\overrightarrow{OM}\| \|\vec{e_y}\| \cos(\beta)$$

$$\beta = \frac{\pi}{2} - \theta \Rightarrow \quad y = \underbrace{\|\overrightarrow{OM}\|}_{r} \underbrace{\|\vec{e_y}\|}_{1} \underbrace{\cos\left(\frac{\pi}{2} - \theta\right)}_{\sin\theta}$$

$$y = R\sin\theta.$$

Application 2



$(\vec{e_r}, \vec{e_\theta})$: base orthonormée.

angle $(\vec{e_y}, \vec{e_\theta}) = ?$

$\vec{e_r}$ dans la base $(\vec{e_x}, \vec{e_y})$?

$\vec{e_\theta}$ // ?

$$\vec{e_r} = \cos\theta\, \vec{e_x} + \overbrace{\sin\theta}^{\cos(\frac{\pi}{2}-\theta)}\, \vec{e_y}$$

$$\vec{e_\theta} = -\sin\theta\, \vec{e_x} + \cos\theta\, \vec{e_y}$$

Plus formellement :

$$\vec{e_r} = \underbrace{(\vec{e_r}\cdot\vec{e_x})}_{\underbrace{\|\vec{e_r}\|}_{1}\underbrace{\|\vec{e_x}\|}_{1}\cos\theta}\vec{e_x} + \underbrace{(\vec{e_r}\cdot\vec{e_y})}_{\underbrace{\|\vec{e_r}\|}_{1}\underbrace{\|\vec{e_y}\|}_{1}\underbrace{\cos(\frac{\pi}{2}-\theta)}_{\sin\theta}}\vec{e_y}$$

idem pour $\vec{e_\theta}$